



Lecture 9

Material and Homework



Read:

Chapter 15 Yariv and Yeh

Chapter 16 Yariv and Yeh

Carrier Density Modulation

- Assume the current driving $I(t)$, the photon density $P(t)^\dagger$, and the carrier density $N(t)$ in a semiconductor laser can be represented by the bias point (I_0, P_0) and a small signal sinusoidal modulation term.

$$I(t) = I_0 + i_1(\omega_m)e^{i\omega_m t}$$

$$P(t) = P_0 + p_1(\omega_m)e^{i\omega_m t}$$

$$N(t) = N_0 + n_1(\omega_m)e^{i\omega_m t}$$

† In previous lectures we have used S to represent the photon density, but the text changes notation.

- Taking the Fourier Transform of the carrier density rate equation, assuming sinusoidal modulation at ω_m , and separating the DC and AC small signal parts into two sets of equations:

DC Steady State Component

$$0 = \frac{I_0}{Vq} - \frac{N_0}{\tau} - A(N_0 - N_{tr})P_0$$

$$0 = A(N_0 - N_{tr})P_0\Gamma_a - \frac{P_0}{\tau_p}$$

AC Small Signal Component

$$-i\omega_m n_1 = -\frac{i}{Vq} + \left(\frac{1}{\tau} + AP_0\right)n_1 + \frac{1}{\tau_p\Gamma_a}p_1$$

$$i\omega_m p_1 = -AP_0\Gamma_a n_1$$

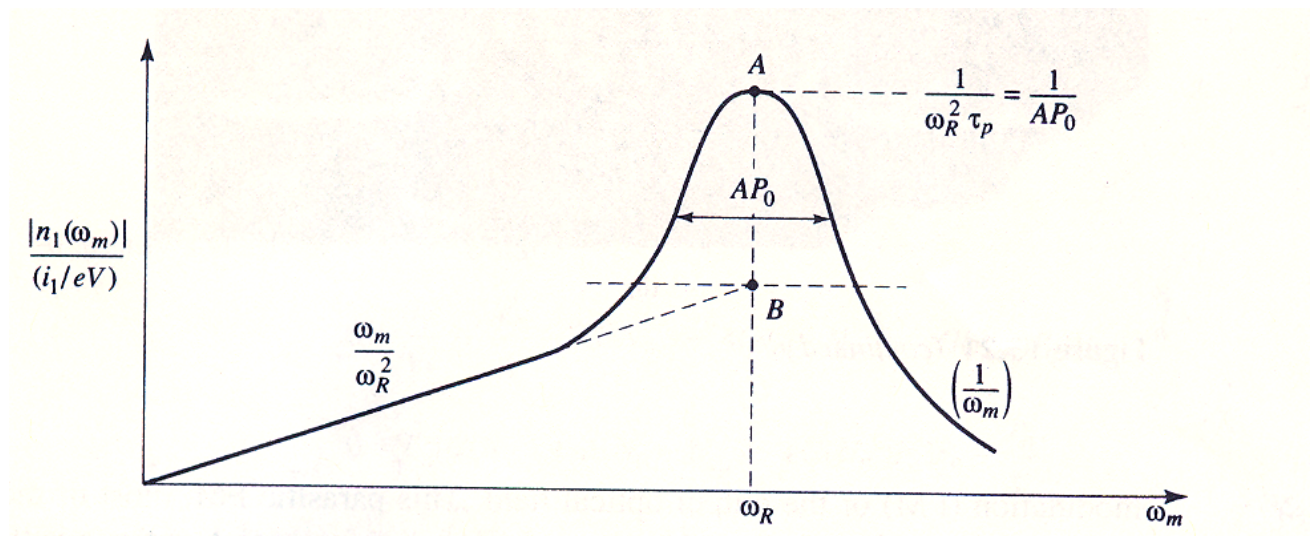
$$n_1(\omega_m) = -i\left(\frac{i_1}{Vq}\right) \frac{\omega_m}{\omega_m^2 - \frac{AP_0}{\tau_p} - i\omega_m\left(\frac{1}{\tau} + AP_0\right)}$$

Carrier Density Modulation

- Plotting the magnitude change in n_1 as a function of ω_m , we see the relaxation oscillation frequency introduced in the last lecture.

$$n_1(\omega_m) = -i \left(\frac{i_1}{Vq} \right) \frac{\omega_m}{\omega_m^2 - \frac{AP_0}{\tau_p} - i\omega_m \left(\frac{1}{\tau} + AP_0 \right)}$$

$$\omega_R = \sqrt{\frac{AP_0}{\tau_p} - \frac{1}{2} \left(\frac{1}{\tau} + AP_0 \right)^2}$$



Gain Suppression (Non-Linear Gain)

- Previously we talked about different effects that reduce the overall material gain.
- Yariv and Yeh differentiate between the following
 - **Gain Saturation:** Drop in gain population N due to increase in photon density
 - **Gain Supression:** Total carrier density N is constant but distribution of carriers as a function of energy or momentum changes with photon density. Former is called **Dynamic Carrier Heating (DCH)** and the later **Spectral Hole Burning (SHB)**.
- Rewriting the optical gain constant to include the **gain supression factor** ϵ by performing a Taylor Series Expansion about $(N=N_{th}, P_0)$

$$G(N, P) = G(N)(1 - \epsilon P) \approx G(N_{th}) + A(N - N_{th}) - \epsilon G(N_{th})P$$

Gain Suppression

- Re-writing the small signal photon density modulation term to include gain suppression

$$p_1(\omega_m) = \frac{-\Gamma_a A P_0 \left(\frac{i_1}{Vq} \right)}{\omega_m^2 - i\omega_m \left(\frac{1}{\tau} + A P_0 + \frac{\varepsilon P_0}{\tau_p} \right) - \left(\frac{A P_0}{\tau_p} + \frac{\varepsilon P_0}{\tau \tau_p} \right)}$$

- Example:

$$P_0 = 1.2 \times 10^{21} \text{ photons/m}^3$$

$$A = 2 \times 10^{-12} \text{ m}^3/\text{s}$$

$$\tau_p = 10^{-12} \text{ s}$$

$$\tau = 10^{-9} \text{ s}$$

$$\varepsilon = 10^{-23} \text{ m}^3$$

$$\frac{\varepsilon P_0}{\tau_p} = 1.2 \times 10^{10} \text{ s}^{-1} \square A P_0$$

Gain Suppression

- Under the operating condition that $\epsilon P_0 / \tau_p \gg AP_0$

$$\omega_R \approx \sqrt{\frac{AP_0}{\tau_p} - \frac{\epsilon^2 P_0^2}{2\tau_p^2}}$$

- So considering gain suppression, P_0 and ω_R reach maximum values at

$$(P_0)_{\max} = \frac{A\tau_p}{\epsilon^2}$$

$$(\omega_R)_{\max} = \frac{A}{\sqrt{2}\epsilon}$$

- Using the values from the previous example

$$(P_0)_{\max} = 2 \times 10^{22} \text{ photons/m}^3$$

$$(f_R)_{\max} = \left(\frac{\omega_R}{2\pi} \right)_{\max} = 2.24 \times 10^{10} \text{ Hz}$$

Laser Frequency Chirping

- Coupled to carrier density modulation ($n_l(\omega_m)$) via Kramers-Kronig, is frequency modulation via phase modulation.
- We define time varying frequency (modulation) as “**Chirp**”
- Solving the frequency domain rate equations for $n_l(\omega_m)$ including gain suppression and converting back to the time domain (by replacing $i\omega_m$ with d/dt) and defining $N(t) = N_0 + \Delta N(t)$

$$n_1(\omega_m) = \frac{\left(i\omega_m + \frac{\varepsilon P_0}{\tau_p} \right)}{\Gamma A P_0} p_1(\omega_m)$$
$$\Delta N(t) = \frac{1}{\Gamma A} \left(\frac{1}{P_0} \frac{dP_0}{dt} + \frac{\varepsilon}{\tau_p} \Delta P(t) \right)$$

Laser Frequency Chirping

- The complex refractive index of a gain medium can be used to derive the **Henry α -factor** and the resulting change in phase $\Delta n_0'$ resulting from a carrier density change $\Delta N(t)$

$$n_0(t) = n_0'(t) - i n_0''(t)$$

$$\Delta n_0'' = -\frac{n_0'}{4\pi\nu} A \Delta N(t)$$

$$\alpha = \frac{\Delta n_0'}{\Delta n_0''} = \frac{\frac{dn}{dN}}{\frac{dg}{dN}}$$

$$\Delta n_0' = -\frac{\alpha n_0' A}{4\pi\nu} \Delta N(t)$$

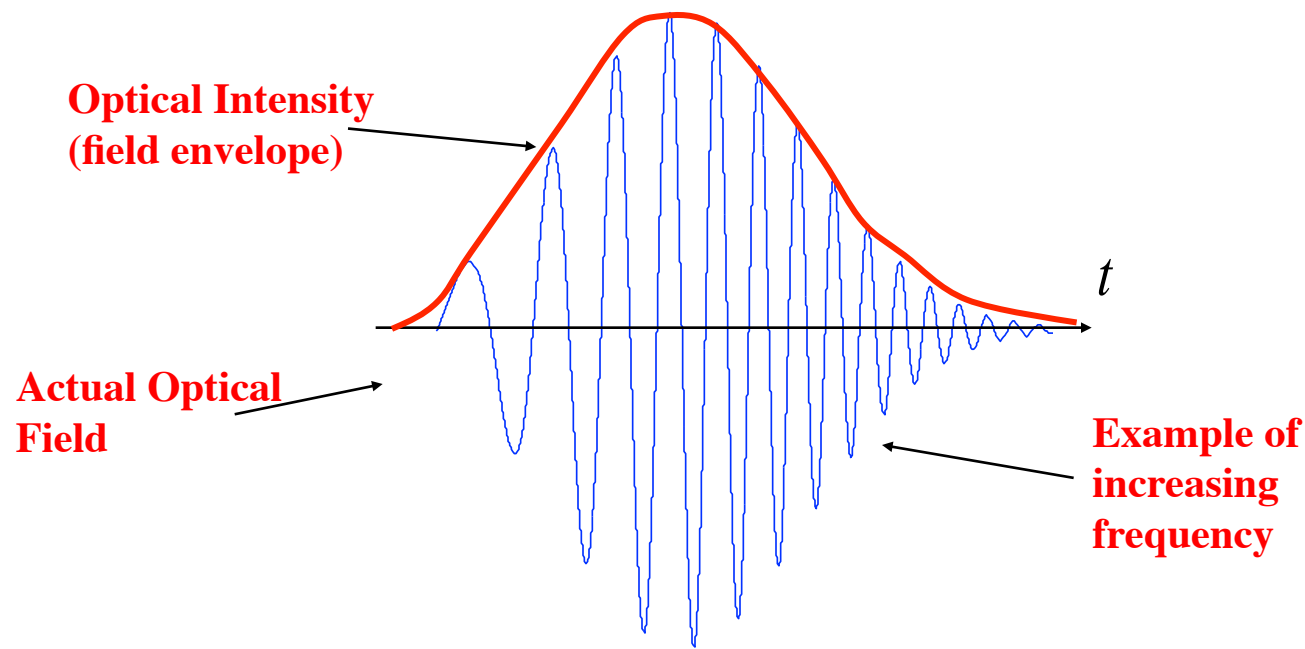
- The change in index via carrier density modulation causes the laser frequency to change from its unperturbed value

$$\frac{\Delta\nu}{\nu} = -\frac{\Delta n_0'}{n_0'} \Gamma_a = \frac{\alpha \Gamma_a A}{4\pi\nu} \Delta N(t)$$

$$\Delta\nu(t) = \frac{\alpha}{4\pi} \left(\frac{1}{P_0} \frac{dP}{dt} + \frac{\varepsilon}{\tau_p} \Delta P(t) \right)$$

Laser Chirp

- Another important parameter is the laser frequency chirp (frequency shift)
- Chirp will limit the bit-rate-distance product that a link can support
- Chirp occurs when directly driving a laser, the change in carrier density changes the effective index of refraction, and thus the oscillation optical frequency
 - This can be interpreted as a bit-synchronous phase or frequency modulation



SC Laser direct modulation

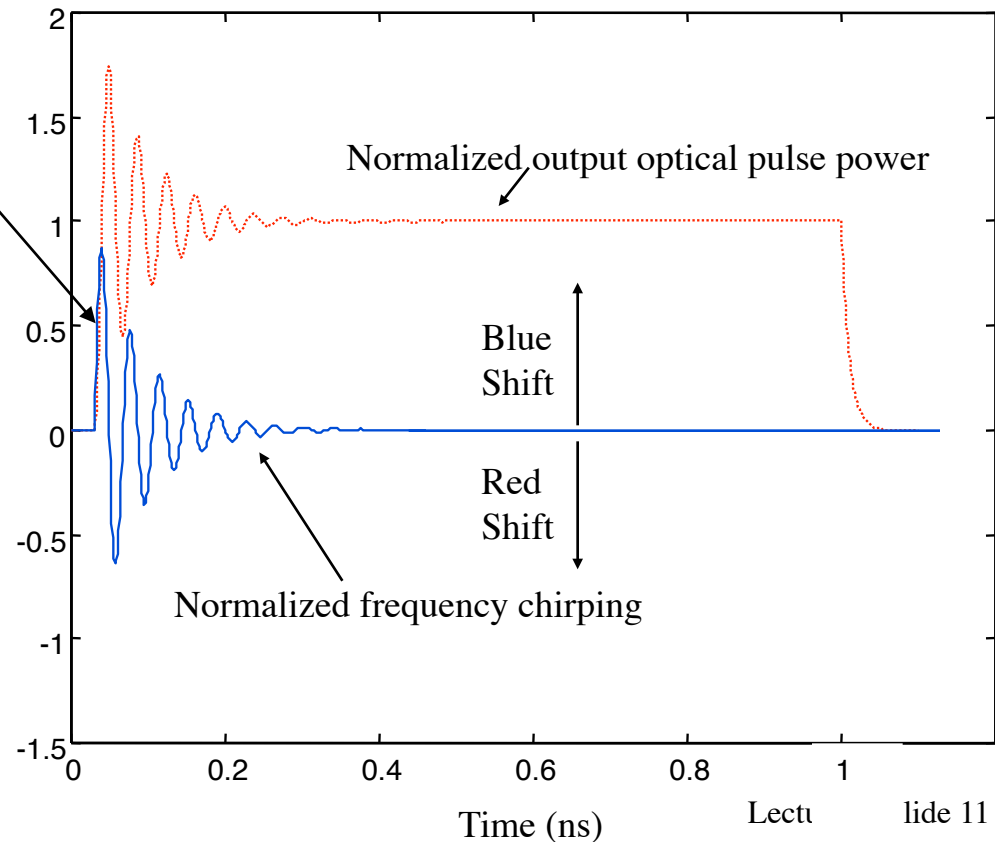
➡ As the laser current is changed between the low and high states, the laser carrier density changes and there is a resulting time dependent phase change.

➡ The time dependent phase changes leads to an instant frequency shift called **frequency chirp**.

$$\Delta\nu(t) = -\frac{\alpha}{4\pi} \left(\underbrace{\frac{1}{P} \frac{dP}{dt}}_{(1)} + \underbrace{\frac{2\Gamma\varepsilon}{V\eta h\nu} \Delta P(t)}_{(2)} \right)$$

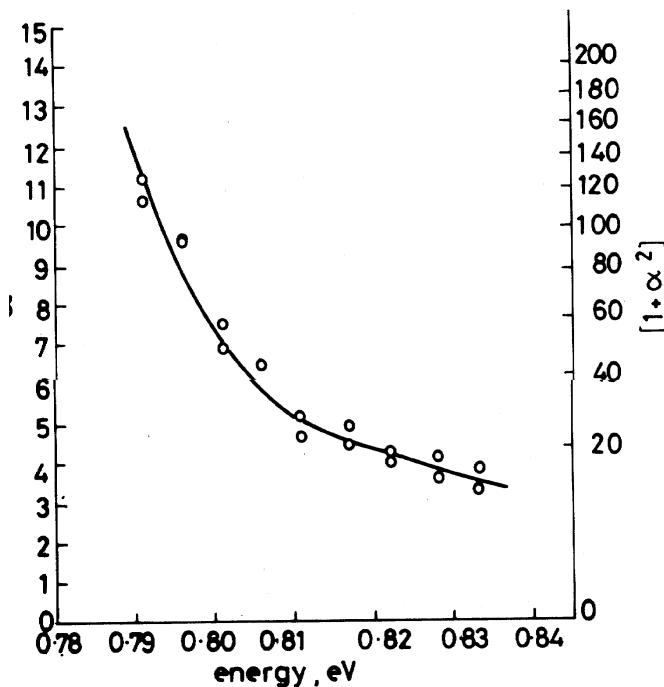
(1) Dynamic chirp: wavelength shift associated with on-off modulation

(2) Adiabatic chirp: Steady-state emission frequency difference between on and off states



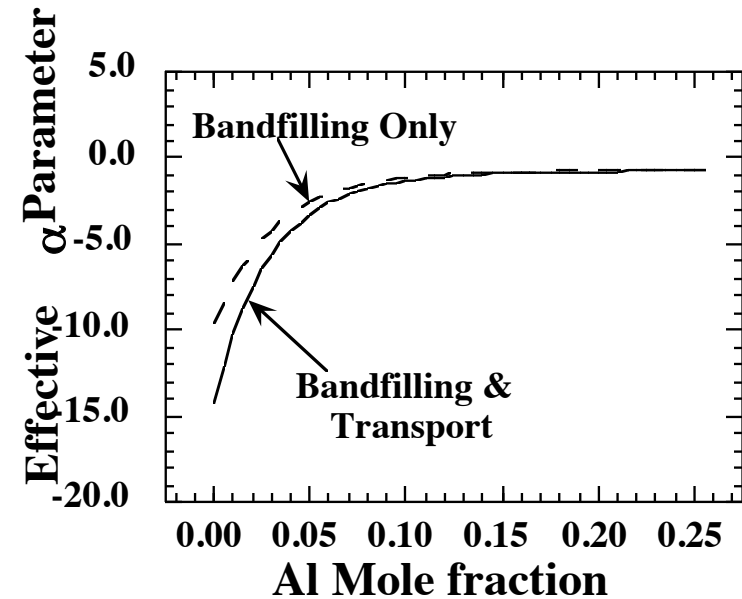
Chirp

The linewidth enhancement factor changes with wavelength, and can also depend on the structure



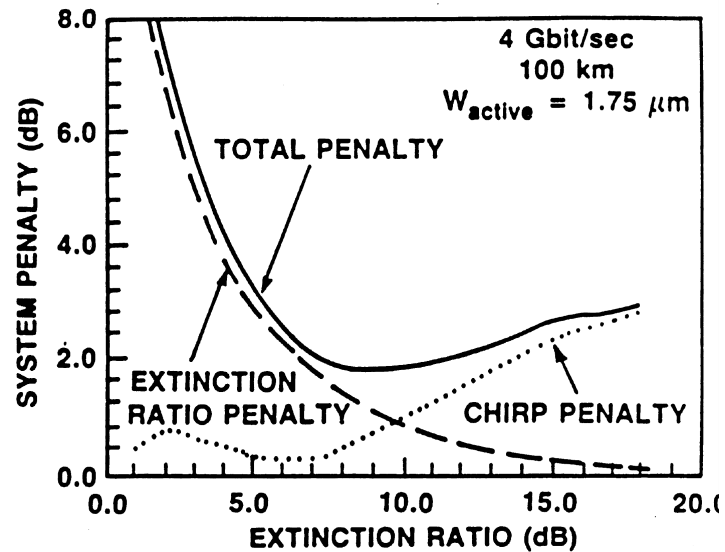
L.D. Westbrook, Electron. Lett., vol. 21, no. 22, 1018 (1984)

R. Nagarajan, J. Quantum Electronics, vol. 29, no. 6, 1601 (1993)

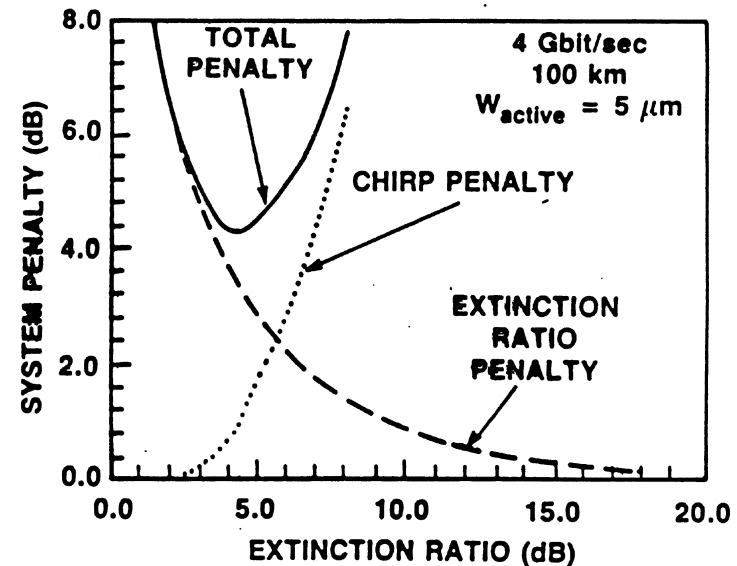


Chirp

Low chirp laser is a requirement to achieve the full potential of an optical communication system



DCPBH Laser



Ridge Waveguide Laser

P.J. Corvini et al., J. Lightwave Technol., vol. LT-5, 1591 (1987)